MATTER AND ENERGY IN LATE 19TH CENTURY BRITISH ELECTROMAGNETISM

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ABSTRACT. In Maxwell’s Treatise, energy was located sometimes inside conducting wires and sometimes outside the conducting wires, in the medium. Maxwell himself stated that the second representation was consistent with his general natural philosophy. Poynting developed this issue and stressed both the storage of energy in the medium and the transference of energy through the medium. Heaviside tried to give a better mathematical foundation to Maxwell’s natural philosophy and opposed Hertz’s attempt to give a formal interpretation of electromagnetism. At the same time, Larmor developed a monistic world view, where aether, electric charge and matter stemmed from the same physical root. J.J. Thomson, in his theoretical researches, represented the electromagnetic radiation as a discrete flow of Faraday’s tubes of force: radiation could be as discrete as matter.

In a book written some years ago, B.J. Hunt, referring to energy and its location in Maxwell’s Treatise, stressed two different representations: energy located in conductors, or energy located in dielectrics.¹ I claim that when Maxwell dressed the suit of the mathematical physicist he handled both representations of energy but when he dressed the suit of the theoretical physicist he chose the second representation: energy spread throughout the medium, aether or matter as well.²

The mathematical expressions corresponding to the first representation were
\[ \int\int\int \left( \frac{1}{2} \mathbf{A} \cdot \mathbf{C} \right) d\mathbf{v} \] for magnetic (electrokinetic) energy and
\[ \int\int\int \left( \frac{1}{2} \rho \Phi \right) d\mathbf{v} \] for electric energy; the mathematical expressions corresponding to the second representation

¹ Cf. Hunt B.J. 1991, pp. 109-10: “On the one hand, the energy of a current could be expressed as the integral over space of \( \mathbf{A} \cdot \mathbf{C} \), the product of the vector potential and the current density. This formula located all the energy of a steady current within the wire, since there was no current outside it. [...] But the energy of the current could also be expressed as \( 1/2 \mu H^2 \), where \( \mu \) is the permeability and \( H \) the magnetic force. When integrated over all space, this expression gives the same value for the total energy of the current as the \( \mathbf{A} \cdot \mathbf{C} \) form, simply in consequence of the defined mathematical relations among \( A \), \( C \) and \( H \). It locates the energy very differently, however: according to the \( 1/2 \mu H^2 \) formula, almost all the energy is outside the wire, in the surrounding magnetic field.”

² Cf. D’Agostino S. 2000, p. xi: “It is true that theoretical physics was mainly a creation of turn-of-the century German physics, where it received full institutional recognition, but it is also undeniable that outstanding physicists in other European countries, namely Ampère, Fourier and Maxwell, also had an important part in creation.”
were \[ \iint \left( \frac{1}{2} \mu \mathbf{H}^2 \right) \, d\mathbf{v} \] for magnetic energy and \[ \iint \left( \frac{1}{2} \varepsilon \mathbf{E}^2 \right) \, d\mathbf{v} \] for electric energy, where the integrals were over all space.

This query about energy calls forth theoretical models for matter and energy in late nineteenth century British electromagnetism. Starting from the concept of Faraday’s lines of force, though submitted to a deep reinterpretation, Maxwell had begun to build his electromagnetic theory. From his first papers till the *Treatise* and beyond, we could follow the history of a theoretical effort of putting mechanics and electromagnetism together. He tried to set a bridge between them but he thought to have not managed to accomplish it in the course of his life. The link between mechanics and electromagnetism showed to be a very problematic link: in late nineteenth century, part of the then not so wide community of physicists undertook the demanding task of exploring the boundaries between the two regions of physics.\(^3\) I would like to try to throw a spot of light in particular on the transformations undergone by the concepts of matter and energy in that borderline region. My subject is limited both in space and in time: the last years of nineteenth century and that part of scientific community living in Great Britain. I claim that the deep transformations taking place in physics of the first decades of twentieth century have hidden roots in attempts and ideas which was boiling in the melting pot of late nineteenth century theoretical physics. Present physics textbooks, even the advanced ones, do not take into account this theoretical heritage; I would like to bring a slight contribution to the comprehension of this hidden tradition.

In late nineteenth century theoretical debates, both matter and energy exchanged their traditional properties: in some theoretical contributions, matter underwent a sort of dematerialisation as well as energy underwent a sort of materialisation. Some theoretical models broke the borderline between continuous and discrete models, giving rise to a conceptual tension between the properties traditionally attributed to the concept of mass and the properties traditionally attributed to the more recent concept of energy or to the older concept of force.\(^4\) At the same time, the theoretical models under consideration contributed to build the first borderline between traditional mathematical physics and emerging theoretical physics. In the background there were even two more general issues: on one hand, the struggle between macroscopic descriptions of physical phenomena and microscopic description in terms of invisible entities; on the other, the struggle between contiguous actions and at-a-distance-actions. Eventually, even the often quoted conceptual gap between the traditions of German and British physics

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3 I cannot agree with Buchwald statement that, apart from some theoretician like Larmor and Lorentz, the community of physicists was not interested in problems at the borderline between mechanics and electromagnetism. Cf. Buchwald J.Z. 1985, p. xi: “The reason for this is quite simple: with the exception of Lorentz, few physicists either in Britain or on the Continent were actively interested in pursuing this problem until c. 1900 […]”. The appraisal of Harman is quite different. Cf. Harman P.M. 1982, p. 116: “When Hertz discussed the electrodynamics of moving bodies in 1890, the problems of ether drag and of the connection between ether and matter were under active consideration by physicists.” For instance, O. Lodge performed experiments to check the supposed aether drag, between 1890 and 1893: cf. Hunt B.J. 1991, p. 192. One of the problems, both theoretical and technical, placed at the borderline between mechanics and electromagnetism involved unipolar induction, namely the electric current stemming from the rotation of magnets around its own axes. Theoretical physicists and engineers, both in Great Britain and on the Continent, wondered whether the magnetic lines of force did rotate together with the magnet or not. For the debate in second half of nineteenth century, cf. Miller A.I. 1981, pp. 155-71.

4 Cf. Harman P.M. 1982, p. 43: “Helmholtz was careful to specify his dual use of the term ‘force’ in his mathematical argument, to denote Newtonian central forces of attraction and repulsion as well as the energy quantities of living force and tensional force; and he was later quick to point out that ‘living force’ and ‘tensional force’ were synonymous with the ‘energy’ terms introduced in the 1850s.”
transformed in a sort of cross-fertilization, as shown by the debate on the new entities recently appeared on the stage of physics (both in experimental and in theoretical researches): cathode rays, ions and electrons.

Following those debates allow us to meet the last generation of physicist who were proud of being natural philosophers; after some years, even the word natural philosophy will have been appeared unsuitable and puzzling for the next generation of physicists. In the course of the twentieth century, theoretical physicists apparently denied their primitive link with that long-lasting tradition. The generation of J.J. Thomson and J. Larmor, who gave its original contribution in the late nineteenth century, was the last generation of natural philosophers and, at the same time, was the first generation who dared crossing those borders. They opened a way. They produced the early contamination between theoretical models of matter and theoretical models of energy, as well as between the already quoted discrete and continuous models, both for matter and energy. The milestone papers the young Einstein wrote in 1905, in particular those on the inertia of energy and on his “heuristischen Gesichtspunkt” on radiation, could appear a bit less astonishing if only we got a look on electromagnetism of the last fifteen years of nineteenth century.5

I would like to stress that I am not thinking in terms of direct influence of something similar: the general theoretical reference frames of J.J. Thomson and Larmor were different from Einstein’s. Even the answer they gave were different, but the questions were the same: it had been just those last generation of natural philosophers who had displayed those questions on the stage of physics.

A long-lasting and widespread standard interpretation has associated to British scientists the flag of contiguous action and to German, or in general Continental, scientists the flag of action-at-a-distance. About electromagnetic theories, the same standard interpretation has associated continuous models of matter and electricity to British and discrete models of matter to Germans. Following this standard interpretation, J.J. Thomson was forced to accept a discrete model of matter and electric charge only after his and other’s 1897 experiments on cathode rays, as well as Larmor was forced to built a theoretical model in terms of electrons only after the defeat of his 1893 continuous model. The fact is J.J. Thomson had tried to reason in terms of discrete models of matter and energy long before 1897 experiments on cathode rays and – even though it could appear even more strange - from a pure theoretical point of view.6 At the same time, Larmor was handling both continuous and discrete models of matter and electricity already in the eighties of the nineteenth century. Both them, in the eighties, undertook a theoretical dialog with Helmholtz’s theories as well as with Maxwell’s. Moreover, in J.J. Thomson and Larmor’s theories we find an original integration between two different British traditions: Maxwell’s electromagnetism and W. Thomson’s aethereal theory of matter. At the same time, they went beyond those traditions.

5 Cf. Einstein A. 1905a, p. 132. “Zwischen den theoretischen Vorstellungen, welche sich die Physiker über die Gase und andere ponderable Körper gebildet haben, und der Maschwellschen Theorie der elektromagnetischen Prozesse im sogenannten leeren Raume besteht ein tiefgreifender formaler Unterschied. […] Er scheint mir nun in der Tat, […] daß die Energie des Lichtes diskontinuierlich im Raume verteilt sei.”

6 Cf. Falconer I. 1887, p. 254: “The cathode rays experiments in 1897 were not the origin of the corpuscle hypothesis; instead they acted as a focus around which Thomson synthesized ideas he had previously developed.” I agree with Falconer and disagree with Navarro, when he states that J.J. Thomson was “the discoverer of the first discrete subatomic particle”, in spite of his faith in “metaphysical continuity of nature” or, in other terms, “his deep belief in the ultimate continuity of matter”. In addition, he was not the only discoverer and even the word discoverer could be questionable.
Larmor explored hypothetical subatomic structures of matter and J.J. Thomson outlined discontinuous structures for the electromagnetic field. They represented a vanguard: they actually pushed forward the boundaries of theoretical physics. In accordance with what is generally believed by physicists, aether theories, as those of J.J. Thomson and Larmor, represented the rearguard of physics at the turn of twentieth century. I claim that some years before that turn, they represented in some way a vanguard, ready to cross the borders between traditional sections of physics as well as to break the borders between the traditional theoretical models for matter and energy.

Maxwell’s electromagnetic theory had challenged previous Continental images of “electric charge”, by representing it as a sort of side-effect of inhomogeneous distribution of potential energy through aether or matter. Some difficulties in explaining conductivity brought J.J. Thomson and Larmor, although in a different way, to develop a non-Maxwellian conception, both dynamical and substantial, of electric charge. At the same time, first in Great Britain and then on Continent, some scientists began to conceive a sort of substantialisation of energy. This was the result of researchers who claimed to be Maxwellians and to follow Maxwell’s conceptual tracks but actually explored new path and came to new lands. J.H. Poynting shifted the attention from Maxwell’s “electric displacement” to the transfer of electromagnetic energy through space and time: he thought he had found in Faraday tubes of force the suitable prop for energy motion. He even thought he had achieved a unified explanation both for induction and conduction in terms of tubes of force. O. Lodge attributed a definite substantial meaning to the concept of Poynting flux of energy and assumed a close similarity between matter and energy. This extreme conception was not shared by the other members of British scientific community and explicitly criticized by another Maxwellian, O. Heaviside. The latter developed

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7 Cf. Maxwell J.C. 1881, pp. 62-3, 66. It seems to me that Darrigol has suitably highlighted Maxwell’s conception of electric charge and electric current in the context of his theory. On one hand, he acknowledges the plurality of Maxwell’s conceptions; on the other hand, he singles out a “core” and a “periphery”, namely the conception consistent with his general theoretical framework and some auxiliary conceptions, devised by Maxwell to explain some specific class of phenomena. Cf. Darrigol O. 2000, p. 174: “The core was essentially macroscopic, in the sense that the basic concept of field, charge and current had a macroscopic meaning. It treated matter and ether as a single continuous medium with variable macroscopic properties (specific inductive capacity, magnetic permeability and conductivity). And avoided speculations on ether models and matter molecules. At the periphery, Maxwell recognized the need for a more detailed picture of the connection between ether and matter. He tried three different strategies. For magnetisation, he modified his theory to integrate molecular assumptions; for electrolysis, he proposed a temporary ionic theory that contradicted his general concept of the electric current; for the Faraday effect, his method was essentially based on a phenomenological modification of the optical Lagrangian, although he invoked a deeper molecular mechanism”. We could say that the conceptual tension between macroscopic and microscopic models was “inaugurated” by Maxwell himself, as well as the first attempts to solve this tension. Cf. Darrigol O. 2000, p. 176.

8 W. Thomson had been continuing to explore the possible connections among aether, matter and electricity. In 1889 he envisaged electric charge as a bridge between aether and matter, although he claimed he was expressing a pure speculation, placed far beyond “the limited present horizon of physical science, than a reality of nature.” Cf. Thomson W. 1889, in Thomson W., Papers, vol. 3, 1890, p. 465.

9 Cf. Poynting J.H. 1885, p. 278: “The hypothesis I propose is that the tubes move in upon the wire, their places being supplied by fresh tubes sent out from the seat of the so-called electromotive force. The change in the point of view involved in this hypothesis consists chiefly in this, that induction is regarded as being propagated sideways rather that along the tubes or lines of induction.”

10 Cf. Lodge O. 1885, p. 482: “On the new plan we may label a bit of energy and trace its motion and change of form, just as we ticket a piece of matter so as to identify it in other places under other conditions; and the route of the energy may be discussed with the same certainty that its existence was continuous as would be felt in discussing the route of some lost luggage which has turned up at a distant station in however battered and transformed a condition.”
Maxwell’s model of a continuous medium, composed in part of ordinary matter and in part of aether, where aether and matter were marked only by different value of the typical constant of an elastic medium: density and elasticity. Energy was spread with continuity throughout the medium and the electric charge was an effect of the distribution of the “electric displacement” in the passage from conductive matter to dielectric matter or aether.\textsuperscript{11}

J.J. Thomson shared Poynting’s firm belief that the concept of “electric displacement” could be misleading, as well as his attempt of going back to Faraday’s tubes of force. Thomson tried a reinterpretation of the equations for the electromagnetic fields $E, D, H, B$, just starting from Faraday’s tubes and, at the end, arrived at a discrete theoretical model for matter, energy and electricity. Energy, placed both in the tubes of force and in the motion of tubes of force, spread and propagated by discrete units, in accordance with a theoretical model quite different from Heaviside’s. In the nineties, Larmor developed a different theoretical model, where discrete units of matter and electricity stemmed from the continuous structure of aether and fields. Going beyond the specific features of every theory, we can see a deep methodological tension between a phenomenological and mathematical tradition, on one hand, and a more recent theoretical, or generally (natural) philosophical, tradition, on the other. If the latter aspired to a intimate representation and explanation of natural phenomena, the first confined itself to a mere quantitative generalisation and description. If the latter made use of mental pictures and displayed sophisticated concepts and models, the first pointed to facts and equations, following an approach that Boltzmann qualified as “mathematical phenomenology”.\textsuperscript{12}

Both the concept of flux of energy and the concept of aethereal matter, even though formally dismissed by twentieth century quantum theory, at a deeper level suggested some hidden link between matter and energy as well as between the structure of electromagnetic field and elementary corpuscles. In other words, in my opinion, the pretended conceptual gap between physics of early decades of twentieth century and theoretical researches on matter and energy in electromagnetism of late nineteenth century are apparent to some extent. I think that the debates on electric charge, matter and energy which took place in the electromagnetic context in late nineteenth century can be considered as one of the roots which fed the subsequent theoretical physics.\textsuperscript{13}

In 1893 J.J. Thomson published a book, \textit{Recent Researches in Electricity and Magnetism}, whose title-page contained the addendum \textit{INTENDED AS A SEQUEL TO PROFESSOR CLERK-MAXWELL’ TREATISE ON ELECTRICITY AND MAGNETISM}. He presented himself as a Maxwell’s follower and intended to accomplish Maxwell’s scientific enterprise, but he will have led Maxwell’s theory towards a direction we do not

\textsuperscript{11} Cf. Heaviside O. 1893, pp. ix: “Maxwell’s theory is a theory of propagation through a simple medium. Fundamentally it is the ether, but when we pass to a solid or liquid dielectric it is still to be regarded as a simple medium in the same sense, because the only change occurring in the equations is in the value of one or both etheral constants, the permittivity and inductivity – practically only the first.”

\textsuperscript{12} Boltzmann L. 1899, in Boltzmann L. 1974, p. 95: “[...] others felt that physics must henceforth pursue the sole aim of writing down for each series of phenomena, without any hypothesis, model or mechanical explanation, equations from which the course of the phenomena can be quantitatively determined; [...] This is the most extreme form of phenomenology, which I should like to call mathematical, [...].”

\textsuperscript{13} I disagree with Buchwald on the comparison he set between late nineteenth century electromagnetic conceptions and theoretical physics of twentieth century: he stressed a formal difference whereas I stress some deep conceptual analogies. Cf. Buchwald J.Z. 1985, p. 41: “Among the several concepts of the quantum revolution which we are today accustomed to thinking of as fundamentally novel is its insistence that a given portion of energy cannot be precisely localized in either space or time.”
know whether Maxwell would have appreciated or not. In a section named
“Electromagnetic Theory of Light”, Thomson tried to give a more detailed account of
propagation of light in terms of tubes of force. The propagation of a plane wave
could be interpreted as “a bundle of Faraday tubes” moving at right angles to
themselves and producing a magnetic force arranged at right angles to both direction
of tubes and direction of motion.

By supposing the number of tubes issuing from the plane source per unit time to
vary harmonically we arrive at the conception of a divergent wave as a series of
Faraday tubes travelling outwards with the velocity of light. In this case the
places of maximum, zero and minimum electromotive intensity will correspond
respectively to places of maximum, zero and minimum magnetic force.

Starting from Maxwell’s theory of electromagnetic fields as stresses
propagating through a continuous medium, Thomson had arrived to picture fields as a
sea of discrete units carrying energy and momentum. Even the wave theory of light,
then a well established theory, seemed violently shaken by a conception which
echoed more ancient theories, since long time considered old-fashioned and
unreliable.

This view of the Electromagnetic Theory of Light has some of the characteristics
of Newtonian Emission theory; it is not, however, open to the objections to which
that theory was liable, as the things emitted are Faraday tubes, having definite
positions at right angles to the direction of propagation of the light. With such a
structure the light can be polarised, while this could not happen if the things
emitted were small symmetrical particles as on the Newtonian Theory.

This passage sounds particularly interesting because of its reference to long-
term debates and long-term cultural processes deeply rooted in the history of science.
The debate on the nature of light and the clash, continuously renewed, between
continuous models and discrete models was the vivid background of Thomson’s
Recent Researches.

The same conceptual tension between continuous models and discrete models
can be found in Larmor’s hypotheses on matter and electric charge. The pages added
by Larmor, in August 1894, to his thick paper “A Dynamical Theory of the Electric
and Luminiferous Medium” consisted of two sections; the first dealt with “atomic
charges” or “primordial atoms” or “monads”, or “electrons”, a name recently used
by J. Stoney. In some way, the new electrons were different from the previous atomic
charges, for they were placed at a different level in the structure of matter: they were
not atoms but entities more elementary than atoms. Atoms could not be the
elementary building blocks of matter any more: they became complex structures and,
in this structure, electrons were involved. A discrete model of matter and electricity
became necessary even at the atomic level, even though these nuclei consisted of
centres of action of the continuous medium in the medium itself.

The circulation of these nuclei along the circuit of the core would constitute a
vortex which can move about in the medium, without suffering any
hydrodynamic pressural reaction on the circulating nuclei such as might tend to

14 Thomson J.J. 1893, p. 11.
15 Thomson J.J. 1893, p. 42.
16 Thomson J.J. 1893, p. 43.
break it up; the hydrodynamic stability of the vortex, in fact, suffices to hold it together.17

A planetary model and a statistical approach were the main features of the molecule structure which Larmor’s attempted to outline. He thought we should have given up localising the position of the electron time by time and it was better to follow Gauss’ strategy of considering the mass of a planet as “distributed round its orbit”. At any point of the orbit he associated a mass density “inversely proportional to the velocity the planet would have when at that point”.18 Once again he managed to built a bridge between the statistical continuous character and the finer discrete character of the phenomenon of microscopic convection.

Just in same way here, the steady flow of the medium, as distinguished from vibrational effects, is the same as each electron were distributed round its circular orbit, thus forming effectively a vortex-ring, of which however the intensity is subject to variation owing to the action of other system.19

At that stage, however, the model was rather rough and Larmor did not deepen the intimate structure of the atom. It was a task which both Larmor and J.J. Thomson will have pursued, although following different paths, in the next years.

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